CS 173 Fall 2018, Lecture A
Final Exam Preparation Document

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1 Overview

The exam will cover all the material taught during the semester, but *not* the material taught on December 4 or 6. The best preparation is to review the problems from the first two midterm prep documents, and then see this document for the additional material – which is only on dynamic programming and cardinality (including countability and uncountability).

The format of the final exam is going to be mainly multiple choice and short answers. There will also be one or two proofs, one of which is likely to be a proof by strong induction.

Topics that will be on the final from the first two midterms

- Proofs by induction (both strong and weak)
- Proofs by contradiction
- Logic
- Sets
- Relations
- Functions
- Recursively defined functions and sets
- Graphs
- Trees
- Reductions between problems (e.g., using oracles)
- Computational complexity (e.g., NP-hardness, Co-NP, etc.)
- Combinatorial counting
- Big-O

Topics that will be on the final but were not examined in the first two midterms

- Dynamic programming
- Cardinality, countability, uncountability

2 Dynamic Programming

Please review the course materials (lecture notes) for dynamic programming, where we presented DP algorithms for several different problems:

- September 27: Fibonacci numbers and Coin Changing problem
- September 29: Longest Increasing Substring and Longest Increasing Subsequence
- October 2: All Pairs Shortest Paths
- November 27: Two-person games and Recognizing Strings in a Language

On the final exam, you may be asked to restate the DP algorithm for the problem, or run the DP algorithm for that problem on some input and show all the calculations. You may also be asked to design a DP algorithm for a very similar problem, and run it on some sample input.

Example problems:

1. Consider the Coin Changing Problem where the coins you are given have the following values: 4 cents, 7 cents, and 9 cents. Your wish to know if you can make change for 23 cents. Show how you fill out the DP matrix $M[0...23]$ where $M[x]$ is True if and only if you can make change for x using these three coins.

2. Apply the DP algorithm for the Longest Increasing Subsequence problem on input

   $$1, -3, 2, 5, -4, 8, 10, 2, 7, 12, 18, 4, 9, 10, 12, 17, -3, 19, 5, 20, 25, 3$$

   Show the entire matrix calculation and the final answer. What is the longest increasing subsequence for this input?

3. Consider the two-person game where there are two piles of rocks. The two players take turns, and the person who takes the last rock off wins. The rules are: each player must take two rocks off, but can take them off the same pile or off different piles (up to the player which he or she does). Assuming that the starting condition has an even number of total rocks, the game always terminates. Suppose you start with u rocks on one pile and v rocks on the other. Your job is to figure out who has a winning strategy, using dynamic programming.

(a) Show a DP algorithm that will compute all the entries of a matrix M[0...u, 0...v] where $M[x, y]$ is 1 if the first player has a winning strategy on starting condition (x, y) (where x is the number of rocks on the first pile and y is the number of rocks on the second pile). You should assume that $M[0, 0] = 2$.

(b) What entry in the matrix has the answer to the question?

(c) What is the running time to compute the matrix, using dynamic programming?

(d) What is the answer to the problem if $u = 5$ and $v = 9$?

4. Consider the language $L \subseteq \{a, b, c\}^*$ defined by

- $a \in L$
- $b \in L$
- If $x \in L$ then $xc \in L$
- If $x \in L$ then $xac \in L$
- L does not contain any other strings than the ones that can be obtained using these rules.

Answer the following questions:

(a) What are all the strings of length 2 in L?

(b) What are all the strings of length 3 in L?

(c) What are all the strings of length at most 5 in L?

(d) Give a DP algorithm that can solve the following decision problem:
Input: string $x \in \{a, b, c\}^*$
Output: Yes if $x \in L$ and No otherwise

5. Consider the edge-weighted graph $G = (V, E)$ with edge weighting w, defined by:

- $V = \{v_1, v_2, \ldots, v_k\}$
- $E = \{(v_i, v_j) | 1 \leq |i - j| \leq 3\}$
- $w : E \rightarrow Z^+$ is defined by $w(v_i, v_j) = 3 - |i - j|$.

Answer the following questions:

(a) Draw the graph and the edge weights when $k = 10$

(b) Apply the All-Pairs Shortest Path algorithm to this graph when $k = 4$

3 Cardinality, countability, uncountability

Do all the problems from Reading Quiz 10 that have to do with these issues, and explain your answers.

4 Other problems

In addition to the problems from the two midterm prep documents, try the following.

1. Practice writing proofs by strong induction. So, write out a proof by *strong induction* for each of the following statements.

   (a) All finite simple graphs have an even number of vertices that have odd degree.

   (b) For all positive integers n, $\sum_{i=1}^{n} i = n(n+1)/2$.