CS 173, Fall 2015
Examlet 3, B Lecture
Solutions

Problem 2: 15 points. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and recall that $P(S)$ denotes the power set of the set S. Let R be a relation on $P(S) \times P(S)$ defined by $R = \{(x, y) \in P(S) \times P(S) : x \subseteq y\}$.

- Give an element of R (2 pts)
  Answer: $(\{1\}, \{1, 2, 3\})$**.**

- Give a pair x, y of subsets of S where (x, y) is not an element of R (2 pts)
  Answer: $x = \{1\}, y = \{2, 3\}$**.**

- Prove that R is reflexive (3 pts)
  Answer: Let $x \in P(S)$**. Note that** $A \subseteq A$ **for any set** A**, and so** $x \subseteq x$**. Hence,** $(x, x) \in R$**.**

- Prove that R is anti-symmetric (4 pts)
  Long Answer: suppose $(x, y) \in R$ **and** $(y, x) \in R$**. Hence** $x \subseteq y \subseteq x$**. If** $y \neq x$ **then since** $x \subseteq y$**, it must be that** $\exists a \in y \setminus x$**. But then** $y \not\subseteq x$**, contradicting** $(y, x) \in R$**. Hence** $y = x$**.**
  Short answer: suppose $(x, y) \in R$ **and** $(y, x) \in R$**. Hence** $x \subseteq y \subseteq x$**. But then** $x = y$**.**

- Prove that R is transitive (4 pts) **Answer: Suppose** $(x, y) \in R$ **and** $(y, z) \in R$**. Then** $x \subseteq y$ **and** $y \subseteq z$**. Hence** $x \subseteq z$**. Furthermore,** x, y**, and** z **are all subsets of** S**. Hence,** $(x, z) \in R$**.**