CS 173, Fall 2015
Examlet 1, B Lecture

NAME (PRINTED):

Acceptance of Honor code: (Signed):

NETID (e.g. hpotter23, not 314159265):

Circle your discussion:

Fri 11 **Fri 12** **Fri 1** **Fri 2** **Fri 3** **Fri 4**

Problem	1	2	3	1-3
Possible	10	10	10	30
Score				

Total out of 30 points

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Problem 1: 10 points. Let the function $F(n) : Z^+ \to Z^+$ be defined by

- $F(1) = 5$
- $F(n) = 2F(n-1)$ for $n \geq 2$

Prove, using induction, that $F(n) = 5 \times 2^{n-1}$ all $n \geq 1$.

Solution:

We verify that the statement holds true for $n = 1$. Thus, $F(1) = 5$ and $5 \times 2^{1-1} = 5 \times 2^0 = 5 \times 1$. Hence, the base case holds.

Our Inductive Hypothesis (I.H.) is that $\exists K \geq 1$ $s.t.$ $\forall n \in \{1, 2, \ldots, K\}, F(n) = 5 \times 2^{n-1}$. We now show that $F(K+1) = 5 \times 2^K$. Since $K + 1 \geq 2$, $F(K+1) = 2F(K)$, by definition. Then we apply the inductive hypothesis, and get

$$F(K+1) = 2F(K) = 2 \times 5 \times 2^{K-1} = 5 \times 2^K.$$

This is what we wanted to prove.

Since K was arbitrary, the theorem is proven.

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Problem 2: 10 points. Simplify the following expression so that it has no $\rightarrow$s and no parentheses. Explain all your steps.

$$(\neg A \rightarrow \neg B) \rightarrow A$$

Solution.

There are many ways to do this problem, including using truth tables. We will do it by simplifying each part.

We begin by simplifying $(\neg A \rightarrow \neg B)$ (the left hand side of the expression).

We know that $P \rightarrow Q \equiv \neg P \vee Q$, so we obtain

$$(\neg A \rightarrow \neg B) \equiv (A \vee \neg B)$$

Thus,

$$[(\neg A \rightarrow \neg B) \rightarrow A] \equiv [(A \vee \neg B) \rightarrow A]$$

We apply the rule for how to remove the $\rightarrow$ again, and obtain

$$[(A \vee \neg B) \rightarrow A] \equiv [\neg(A \vee \neg B) \vee A]$$

We now have to use De Morgan's laws to remove the parentheses. Note that $\neg(A \vee \neg B) \equiv (\neg A \wedge \neg\neg B) \equiv (\neg A \wedge B)$. Hence, we continue the reduction and have

$$[(A \vee \neg B) \rightarrow A] \equiv [\neg(A \vee \neg B) \vee A] \equiv [(\neg A \wedge B) \vee A]$$

which we write more simply as

$$[(A \vee \neg B) \rightarrow A] \equiv [(\neg A \wedge B) \vee A]$$

We now have to use the distribution property to simplify the right hand side. We obtain

$$[(\neg A \wedge B) \vee A] \equiv [(\neg A \vee A) \wedge (B \vee A)$$

Since $\neg A \vee A \equiv T$, the above expression simplifies to $B \vee A$, which is equivalent to $A \vee B$. Hence, we have shown

$$[(A \vee \neg B) \to A] \equiv (A \vee B)$$

Since the parentheses around $A \vee B$ aren't necessary (and are only there to help you parse the sentence), what we have shown is

$$[(\neg A \to \neg B) \to A] \equiv A \vee B$$

We now check that our answer is correct, because it's easy to make mistakes. If A is true, then the right hand side of the expression $(\neg A \to \neg B) \to A$ is true, which sets the whole expression to true. If A is false, then the right hand side is false, and so the expression is true if and only if the left hand side is false. But since A is false, to make the expression $\neg A \to \neg B$ false, we must have B being true. Hence, we have verified that the expression is true if and only if A is true or B is true, which is what we had derived through simplifying the expressions.

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Problem 3: 10 points. Consider the following 2CNF formula:

$$(\neg A \vee \neg B) \wedge (B \vee C)$$

- Determine whether this formula is a tautology, satisfiable but not a tautology, or not satisfiable (and prove your answer correct). (5pts)

  Solution: the expression is satisfiable but not a tautology. To satisfy it, you can set $A = F$ and $B = C = T$. But it is not a tautology, because setting $A = B = C = F$ makes the expression false.

- Show the directed graph associated to this 2CNF formula (as described in the class presentation). (5 pts)

  Solution: There are three variables so six vertices, and two clauses so four directed edges. The vertices are named after the variables and their negations. The directed edges are:

  $A \rightarrow \neg B$

  $B \rightarrow \neg A$

  $\neg B \rightarrow C$

  $\neg C \rightarrow B$

  If you draw the graph, you will find two vertex-disjoint directed paths: $A \rightarrow \neg B \rightarrow C$ and $\neg C \rightarrow B \rightarrow \neg A$.