1 Practice in writing mathematics

Problem 1: The problem here is to use set builder notation to define the set of functions from the integers to the integers that are not constants.

To do this problem, you have to start by thinking how to define the constant functions. A constant function has the same output on every input. For example, the function defined by $f(x) = 3$ for all $x \in \mathbb{Z}$ is a constant function. On the other hand, the function $f(x) = x \mod 2$ that maps all even integers to 0 and all odd integers to 1 is not a constant function.

You want to define a set that contains only non-constant functions but also contains every non-constant function. Think about both conditions as you try to solve this problem, and think in particular about examples of non-constant functions (like $f(x) = x \mod 2$) and make sure that the function is in the set you define.

One correct solution is:

- $\{f : \mathbb{Z} \to \mathbb{Z} | \exists \{a, b\} \subset \mathbb{Z} \; s.t. \; f(a) \neq f(b)\}$

What this asserts is that every non-constant function has two elements a and b that are mapped to different values.

Here's another correct solution:

- $\{f : \mathbb{Z} \to \mathbb{Z} | \forall a \in \mathbb{Z} \; \exists x \in \mathbb{Z} \; s.t. \; f(x) \neq a\}$

This is the same as saying that every non-constant function doesn't map every integer to the same value.

Here are some incorrect solutions.

- $\{f : \mathbb{Z} \to \mathbb{Z} | \exists a \in \mathbb{Z} \; s.t. \; \forall b \in \mathbb{Z} \; f(a) \neq f(b)\}$

Every function in this set is indeed non-constant, but the set doesn't contain all the non-constant functions. Consider, for example, $f(x) = x \mod 2$. This should be in the set, but it isn't.

- $\{f : \mathbb{Z} \to \mathbb{Z} | \forall \{a, b\} \subseteq \mathbb{Z}, f(a) \neq f(b)\}$

Every function in this set is non-constant, but like the previous example, $f(x) = x \mod 2$ isn't in the set.