Extra homework, due February 21 at 1 PM

Submit this (in PDF format on Moodle, or in hardcopy in class) anytime between now and February 21 at 1 PM. Professor Warnow will grade this homework.

1. Define what it means to say that a function $f : Z^+ \rightarrow Z^+$ is $O(g)$ where $g : Z^+ \rightarrow Z^+$. (Give the formal definition in terms of two constants.)

2. Define what it means for f to be a Karp reduction from problem π to problem π'.

3. What is the class *NP*? (Do *not* use the words "non-deterministic" in your answer.)

4. What does it mean for problem π to be *NP*-hard? (Don't just say π is at least as hard as any problem in *NP*.)

5. Are there any problems in *NP* that can be solved in polynomial time? If so, given an example of such a problem, or else prove no such problem exists.

6. Is it possible for an *NP*-hard problem to be also in *P*?

7. Are there *NP*-hard problems that are not in *NP*? If so, give an example; otherwise, prove no such problem exists.

8. Describe a Karp reduction from 2-colorability to 3-colorability, and explain why it satisfies all the requirements of being a Karp reduction.

9. Draw seven simple graphs $G = (V, E)$, each with no more than six vertices. Then see which of the following properties holds for the graphs.
   - $\forall v \in V \exists w \in V \ s.t. \ (v, w) \in E$
   - $\forall v \in V \exists w \in V \ s.t. \ (v, w) \notin E$
   - $\exists v \in V \exists w \in V \ s.t. \ (v, w) \in E$
   - $\exists v \in V \exists w \in V \ s.t. \ (v, w) \notin E$
   - $\forall v \in V \forall w \in V \setminus \{v\}, (v, w) \in E$
   - $\forall v \in V \forall w \in V \setminus \{v\}, (v, w) \notin E$
   - $\exists v \in V \ s.t. \ \forall w \in V, (v, w) \in E$
   - $\exists v \in V \ s.t. \ \forall w \in V, (v, w) \notin E$
   - $\exists X \subseteq V$ s.t. $|V| \leq 3$ and $\forall v \in V \setminus X$, $\exists w \in X \ s.t. \ (v, w) \in E$
   - $\exists X \subseteq V$ s.t. $|V| \leq 3$ and $\forall v \in V \setminus X$, $\exists w \in X \ s.t. \ (v, w) \notin E$

10. Let us say that vertices v and w are *path-connected* in the graph G if there is a path in G between v and w. Consider the path graph P_n with $n = 100$ (i.e., P_{100} has 100 vertices $v_1, v_2, \ldots, v_{100}$, with $(v_i, v_{i+1}) \in E$ for $i = 1, 2, \ldots, 99$).

- What is the number of vertices in the largest set $X \subseteq V$ so that every two vertices in X are path-connected?
- What is the largest independent set in G?
- What is the largest clique in G?
- What is the smallest dominating set in G?
- What is the minimum number of colors needed to properly color the vertices of G?
- Is P_{100} a tree?
- Is P_{100} Eulerian?
- Is P_{100} Hamiltonian?
- Is P_{100} 2-colorable?
- Is P_{100} connected?

11. Describe the following problems in terms of graphs (i.e., state how you would define the graph, and what graph-theoretic problem you would then need to solve): In each case, assume you know who likes whom in some class (say, Anthropology 100 at Euphoria State University).

    - You want to find the largest number of people in the class who all like each other.
    - You want to find the smallest number of people in the class so that everyone in the class likes at least one of the selected people.
    - You want to find out if you can pair up the people in the class (disjoint pairs) so that every pair of people like each other.
    - You want to find out if you can divide the class into two groups so that everyone within each group likes each other.
    - You want to find the largest number of people in the class that dislike each other.
    - You want to find the smallest number of people you can remove from the class so that everyone left in the class has someone they like left in the class.