CS173 Lecture B, December 8, 2015

Tandy Warnow

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Last class day!

Today's agenda:

- Discuss grades
- Discuss final exam
- Go over last homework
- Review questions for final
- ICES forms (do you have pencils?)

Grades: You have done very well!

Current calculation of final course grade is based on everything that will count *except* Examlet 8 (graded, but not all grades entered into Moodle) and the final exam.
Based on this set of grades, here is the distribution:
Total number students: 164

- A: 45%
- B: 37%
- C: 10%
- D: 6%
- F: 2%

Note this is very top-heavy! More A's than has generally been seen in prior 173 courses! (Typical in past seems to be about 60% between A's and B's; we have 82%.)

Final exam

The final will be largely from the review questions on the class webpage.
Please come to office hours if you have a question about how to solve a problem.
My office hours today are 1-3 PM.

Question 1 on HW #14: solutions

Question 1: Suppose $G = (V, E)$ is a graph and $V_0 \subseteq V$ is both a vertex cover and an independent set. What must be true about G?
Solution: G must be bipartite.
Proof: Let $V_1 = V \setminus V_0$.
We will prove that all edges in G are of the form (a, b) where $a \in V_0$ and $b \in V_1$.
Because V_0 is an independent set, there are no edges between vertices in V_0.
Suppose that $(a, b) \in E$.
Since V_0 is a vertex cover, at least one endpoint of the edge (a, b) is in V_0.
Hence it cannot be that both endpoints are in V_1.
Hence G is bipartite, with bipartition V_0, V_1.

Question 2 on HW 14: solutions

Question 2: What is true for all trees?

- The chromatic number is at most 2 - TRUE
- Every edge in T is a bridge - TRUE
- There is exactly one path between every two vertices in T - TRUE
- If the tree has at least three vertices, then the set of leaves is an independent set - TRUE
- The maximum clique size is at least three - FALSE

Question 3 on HW 14: solutions

Question 3: Consider the set X of all sets of exactly three integers.

- X is finite - FALSE
- X is infinite - TRUE
- X is uncountable - FALSE
- X is countably infinite - TRUE
- $|X| = |\mathbb{N}|$ - TRUE

Question 3 on HW 14: proof that X is infinite

Proof by contradiction: If X were finite, then $\exists n \in \mathbb{N}$ such that $|X| = n$.
So $X = \{S_1, S_2, \ldots, S_n\}$.
Let $p = \max\{x : \exists i, x \in S_i\}$.
Note that $A = \{p+1, p+2, p+3\} \neq S_i$ for any i.
But A is a set of exactly three integers, and so $A \in X$.
This is a contradiction.

Question 3 on HW 14: proof that X is countably infinite

Let A_n be the set of all subsets of three integers drawn from $[-n, n]$.
Note that A_n is finite (you can calculate how large A_n is).
Note that $X = \bigcup_n A_n$. (This is not the disjoint union.)
So X is the union of countably many finite sets.
We know that any such set is countably infinite.
In fact, any set that is the union of countably many countable sets is countable (we did this in class).
Hence, $|X| = |\mathbb{N}|$, by definition of countably infinite.

Question 4 on HW #14

Consider the complete bipartite graph $K_{m,n}$ with $m \geq n \geq 1$. What is the size of a minimum vertex cover?
Answer: n: Suppose $V(K_{m,n}) = V_0 \cup V_1$, with V_0 the m vertices on the left side and V_1 the n vertices on the right side of G.
Then both V_0 and V_1 are vertex covers (every edge has at least one endoint in each of these sets).
V_1 has $n \leq m$ vertices, so we pick V_1.
We will prove that V_1 is a smallest vertex cover, by contradiction.

Question 4 on HW #14

Suppose V' is a smaller vertex cover.
Since $|V'| < n$, V' cannot include all of V_1 nor all of V_0.
Hence $\exists a \in V_0 \setminus V'$ and $\exists b \in V_1 \setminus V'$.
Note that $(a, b) \in E(K_{m,n})$ and yet neither endpoint is in V'.
Therefore V' cannot be a vertex cover.
Therefore V_1 is a minimum vertex cover.

(Question: do you think V_1 must be the unique smallest vertex cover for $K_{m,n}$? It's not true if $m = n$, but what if $m > n$?)

Question 5 on HW #14

Consider the complete bipartite graph $K_{m,n}$ with $m \geq n \geq 1$. What is the size of a largest independent set?

Answer: the answer is m. As for the previous question, let V_0 be the left side with m vertices, and V_1 be the right side with n vertices.
Both V_0 and V_1 are independent sets, but V_0 is at least as large.
Suppose V' is a larger independent set, and so has at least $m+1$ vertices.
Hence V' must contain at least one node $a \in V_0$ and one node $b \in V_1$.
But then $(a, b) \in E(K_{m,n})$, which means V' is not an independent set.

Question: do you think V' is the unique solution? Again if $m = n$ the answer is NO, but what if $m > n$?

Question 6 on HW #14

Consider the complete bipartite graph $K_{m,n}$ with $m \geq n \geq 2$. What is the size of a minimum dominating set?

The answer is 2: take one vertex v from the left side and one vertex w from the right side, so your set is $\{v, w\}$.
Note that any such set is a dominating set. Could we find a smaller dominating set?

Question 6 on HW #14

A smaller set would contain exactly one vertex x. We will show no such set can be a dominating set.
Suppose $x \in V_0$, the left side of the graph with $m \geq 2$ vertices.
Hence $\exists y \in V_0 \setminus \{x\}$.
Note that $(x, y) \notin E(K_{m,n})$, and so $\{x\}$ is not a dominating set.

This is a contradiction.
The same proof would work if $x \in V_1$, since V_1 also has at least two vertices.

Question 6 on HW #14

Additional question: Suppose $m \geq 2$ and $n = 1$. What is the size of a minimum dominating set?

Any other questions?

Good luck!

Thanks for the fun semester - and especially thanks to those of you who came to office hours.
I've enjoyed our one-on-one conversations tremendously.