CS173, Review for Midterm 1

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Review for Midterm 1

Today

Review for Midterm 1

- structure of the exam
- no notes, no extra paper, etc
- midterm prep problems

Structure of exam

- Three proofs (give full details, justify every step that isn't arithmetic!):
  - Proof by weak induction of a theorem (20 pts)
  - Proof by contradiction of the same theorem (20 pts)
  - Proof by strong induction of a different theorem (20 pts)
- Short questions (40 pts)

Things to watch out for in induction proofs

1. Do your base cases!
2. Your Inductive Hypothesis should be stated in terms of some statement P(n) being true for some arbitrary n that is at least as big as the largest base case you looked at
3. Clearly state your Inductive Hypothesis (and label it as such) - and don't have it be what you want to prove!
4. For strong induction, don't confuse your variables with each other (they are not interchangeable)
5. It can be helpful to say what you want to show
6. Justify every step (e.g., every equality sign) that isn't due just to arithmetic
7. At the end, say something like "Because n was arbitrary, this shows that P(n) is true for all n" or "By the principle of induction, this shows that P(n) is true for all n"

More on induction proofs

It can be helpful to say what you want to show:

- For weak induction, you might want to show that $P(n)$ implies $P(n+1)$
- For strong induction you might want to show that $P(1) \wedge P(2) \wedge \ldots \wedge P(n)$ implies $P(n+1)$

Justify every step (e.g., every equality sign) that isn't due just to arithmetic:

- Show where you are using your inductive hypothesis
- Show where you are using the definitions you were given

For strong induction

For strong induction, don't confuse your variables with each other (they are not interchangeable)

For example, suppose you want to prove that $f(n) = 0$ for all $n \in \mathbb{Z}^+$ where f is defined by

- $f(1) = f(2) = 0$
- $f(n) = f(n-2)$ if $n \geq 3$

You say $P(n)$ asserts that $f(n) = 0$, and you check base cases $n = 1, 2$.

Your Inductive Hypothesis is: for some arbitrary $n \geq 2$, $P(k)$ is true for all integers k between 1 and n

You now want to show that $P(n+1)$ is true.

For strong induction

For example, suppose you want to prove that $f(n) = 0$ for all $n \in \mathbb{Z}^+$.

You say $P(n)$ asserts that $f(n) = 0$, and you check base cases $n = 1, 2$.

Your Inductive Hypothesis is: for some arbitrary $n \geq 2$, $P(k)$ is true for all integers k between 1 and n

You now want to show that $P(n+1)$ is true.

Notes:

- You do not say you want to show that $P(k+1)$ is true.
- You do not say $P(k) = 0$
- You do not say $f(n)$ is true
- Your Inductive Hypothesis is not "for some arbitrary $n \geq 3$"
- Your Inductive Hypothesis is not "for some arbitrary $n \geq 2$ and for all k between 1 and n, $P(n)$ is true"

Proof by contradiction

Review September 11 lecture for how to do this.

Two proofs of the same statement

Let A_n be a sequence of sets $n = 1, 2, \ldots$, defined by

- $A_1 = \{100\}$
- $A_n = A_{n-1} \cup \{n + 99\}$ for $n \geq 2$

Find a closed form solution for A_n and prove it correct in two ways: by induction and by contradiction.

Short questions

Do not show your work on these questions - just give your answers!
(No partial credit for this part)

- Logic
- Sets
- Binary relations
- Functions
- Combinatorial counting

Logic questions

For each statement below, say if it is a tautology, satisfiable but not a tautology, or always false?

- $(P \to (\neg P \vee Q)) \wedge \neg Q$
- $(P \to \neg P) \wedge P$
- $P \to (P \vee \neg P)$

Logic questions

Which of the sets S below satisfy the property that $\exists x \in S$ s.t. $\forall y \in S, x \neq y \rightarrow x < y$

- $S = \mathbb{Z}$
- $S = \mathbb{R}^+$
- $S = \mathbb{Z}^+$
- $S = \{-3, 4, 5\}$

Logic questions

Which of the sets S below satisfy the property that $\forall\{x, y\} \subset \mathbb{R}, (x \in S \wedge y > x) \rightarrow y \notin S$?

- $S = \mathbb{Z}$
- $S = \mathbb{R}^+$
- $S = \mathbb{Z}^+$
- $S = \{-3, 4, 5\}$

Functions

1. What is the meaning of a function being $1-1$ (also called "injective")?
2. What is the meaning of a function being onto (also called "surjective")?
3. Consider the set of functions from $\{a, b, c\}$ to the days of the week:
   3.1 How many are there?
   3.2 Are any of these functions injective? (If so, define it.)
   3.3 Are any of these surjective? (If so, define it.)

Binary Relations

Consider the binary relation Q on $\mathbb{Z}$ defined by $(a, b) \in Q$ if and only if $a + b = 6$.

- Give some elements of Q
- Is Q transitive, reflexive, symmetric, anti-symmetric?

Binary Relations

Consider the binary relation T on $\mathbb{P}(\mathbb{Z})$ defined by $(a, b) \in T$ if and only if $a \cap b \neq \emptyset$

- Give some elements of T
- Is T transitive, reflexive, symmetric, anti-symmetric?

During the exam

- Know what discussion section you are in (ADA, ADB, etc.) - you'll have to write that down
- Be on time!
- Do not disturb the other students
- If you have questions, go to the Aisle and wait for a proctor to come to you
- Be honorable.
  - No notes, no cellphones, no cheating of any sort
  - Do not tell other students who haven't yet taken the exam what is on the exam