# CS173
# Dynamic Programming

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# CS 173
# Dynamic Programming Algorithms
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# Today's material

- Dynamic Programming (DP) essentials (review)
- Making change with two coins

# Making change with two coins

Suppose we have two coins – 5 cents and 7 cents. What total values can we make?

- We can't make anything smaller than 5 cents
- We can make 5 cents
- We can't make 6 cents
- We can make 7 cents
- We can't make 8 cents
- We can't make 9 cents
- We can make 10 cents
- We can't make 11 cents
- We can make 12 cents

# Making change with two coins

Suppose we have two coins – 5 cents and 7 cents. What total values can we make?

Can we come up with an algorithm for this?

# Making change with two coins

Suppose we have two coins – 5 cents and 7 cents. What total values can we make?

Let's define a boolean matrix called "Change" where $Change[x]$ is true if and only if we can make change for $x$ cents.

Shall we solve this recursively or by dynamic programming?

# Making change with two coins

We can initialie the first 12 values using what we figured out before (i.e., for $x = 1, 2, \ldots, 12$ we already know the answers).

Now suppose we have figured out the correct values for $Change[x]$ for all $x = 1, 2, \ldots, N$.

How do we figure out the correct value for $Change[x + 1]$?

# Making change with two coins

We can initialie the first 12 values using what we figured out before (i.e., for $x = 1, 2, \ldots, 12$ we already know the answers).

Now suppose we have figured out the correct values for $Change[x]$ for all $x = 1, 2, \ldots, N$.

Suppose we *can* make change for x cents. What would that look like, given that $x > 12$?

- Observation: The change would have to have at least one 5 cent coin or at least one 7 cent coin.

# Making change with two coins

We can initialie the first 12 values using what we figured out before (i.e., for $x = 1, 2, \ldots, 12$ we already know the answers).

Now suppose we have figured out the correct values for $Change[x]$ for all $x = 1, 2, \ldots, N$.

Suppose we *can* make change for x cents. What would that look like, given that $x > 12$?

- Observation: The change would have to have at least one 5 cent coin or at least one 7 cent coin.

So: if $Change[x]$ is true, then at least one of $Change[x - 7]$ and $Change[x - 5]$ is true.

# Making change with two coins

We can initialie the first 12 values using what we figured out before (i.e., for $x = 1, 2, \ldots, 12$ we already know the answers).

Now suppose we have figured out the correct values for $Change[x]$ for all $x = 1, 2, \ldots, N$.

Suppose we could make change for $x - 7$ or $x - 5$ using 5 cent and 7 cent coins.

Then we could definitely make change for $x$ cents by adding one of those coins!

# Making change with two coins

Note that we can always make change for 0 cents (just give no coins).

Putting this together:

- $Change[0]$ is true.
- $Change[x]$ is false for $x = 1, 2, 3, 4$
- If $x \geq 5$, then $Change[x]$ is true if and only if at least one of $Change[x - 7]$ or $Change[x - 5]$ is true.

Also, we can use the calculation to figure out how to give change!

# Making change with two coins

Class assignment:

- Calculate $Change[x]$ for all $x = 1, 2, \ldots, 25$.
- What is the largest value for $x$ for which you can make change?
- Harder: How do you make change for that value for $x$? (Hint: look at how you derived the "true" solution)

# Making change

Questions:

1. How would you solve this problem for two arbitrary valued coins?
2. How would you solve this problem for three arbitrary valued coins?
3. How would you solve the problem of finding the smallest number of coins you need to make change for $x$, assuming you can make change?