CS173, Minimum Spanning Trees

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A **spanning tree** of a connected graph $G = (V, E)$ is a subgraph that includes all the vertices and is a tree.
Minimum Spanning Trees (MST)

- Input: Connected graph $G = (V, E)$ and positive edge weights $w : E \to Z^+$
- Output: Spanning tree $T = (V, E')$ of $G$ that has minimum cost, where $\text{cost}(T) = \sum_{e \in E'} w(e)$
Finding MSTs

Try one of the greedy algorithms on the complete bipartite graph $K_{3,5}$ with $w(v_i, w_j) = i + j$:

- Keep adding the least weight edges (don’t include those that create cycles) - Kruskal’s algorithm
- Keep deleting the most costly edges (don’t delete bridges)
- Grow a spanning tree, adding least costly edge to an unvisited vertex - Prim’s algorithm
Finding MSTs

Running Kruskal’s algorithm on $K_{3,5}$ with weight $w(v_i, w_j) = i + j$: 
Finding MSTs

Why did that method work?